Minimal Problems in Computer Vision

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joint work with

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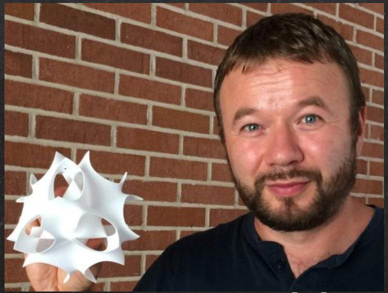

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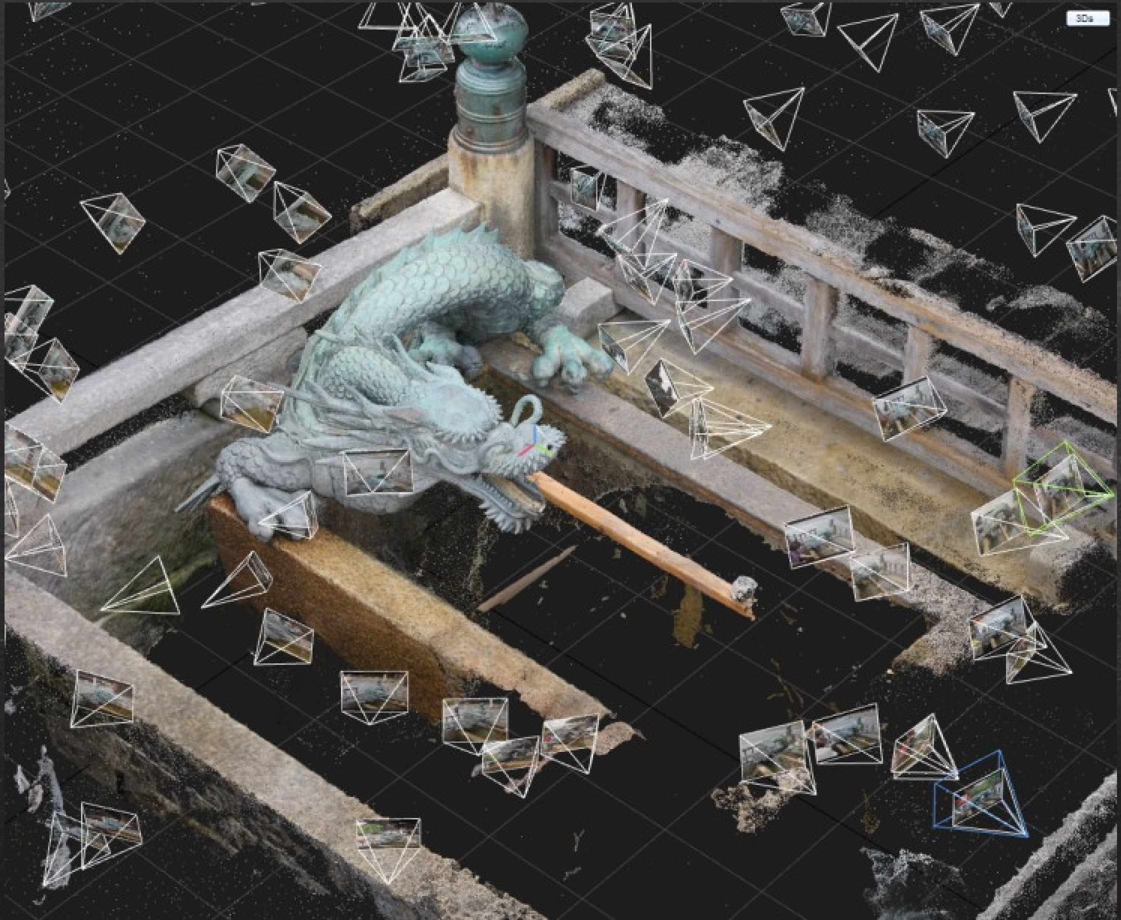
(Georgia Tech)

Tomas Pajdla
(CIIRC CTU in Prague)

Goal:

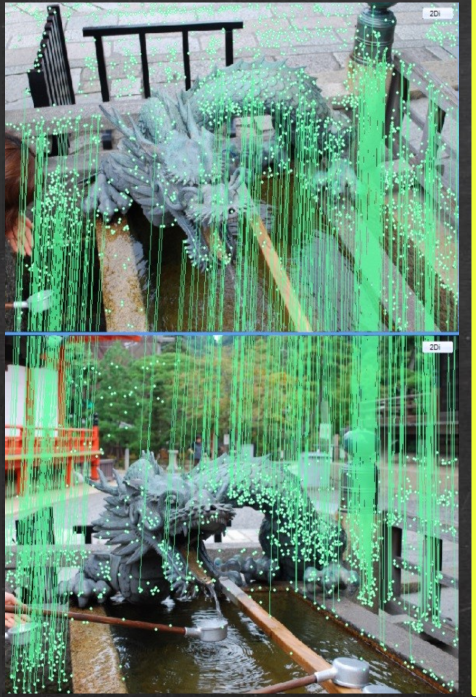
Reconstruct 3D scenes and camera poses from 2D images



3D Reconstruction Pipeline

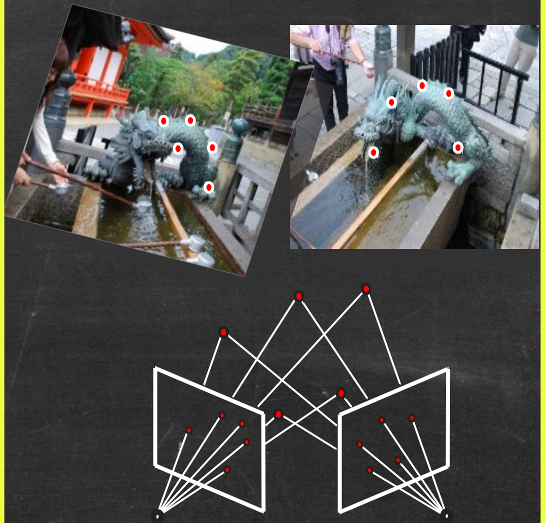
Input images

Image Matching



Identify common points and lines on given images

Camera Geometry



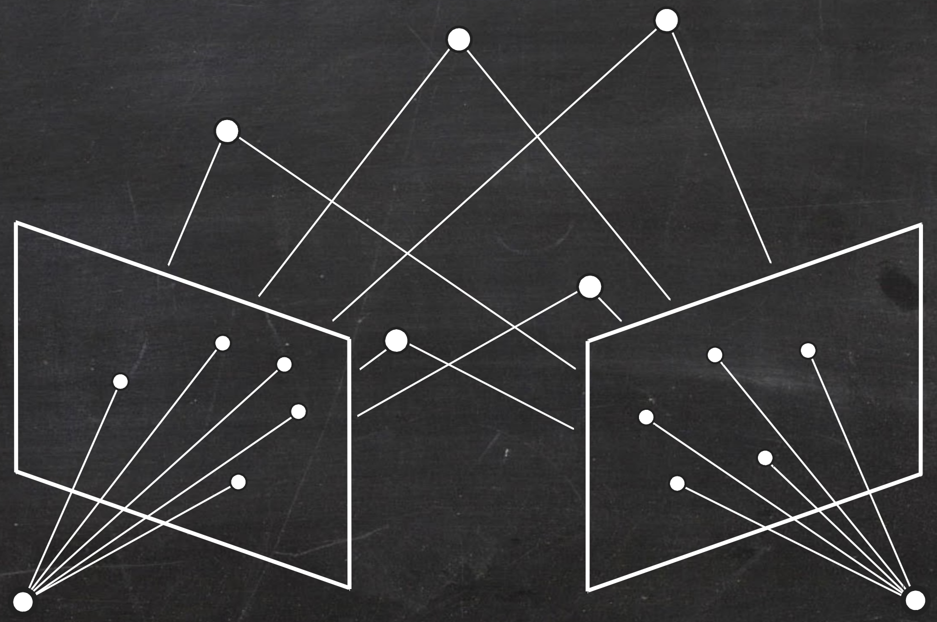
Reconstruct 3D points and lines as well as camera poses

Cameras & points

This is an **algebraic** problem!

Example: The 5-Point Problem

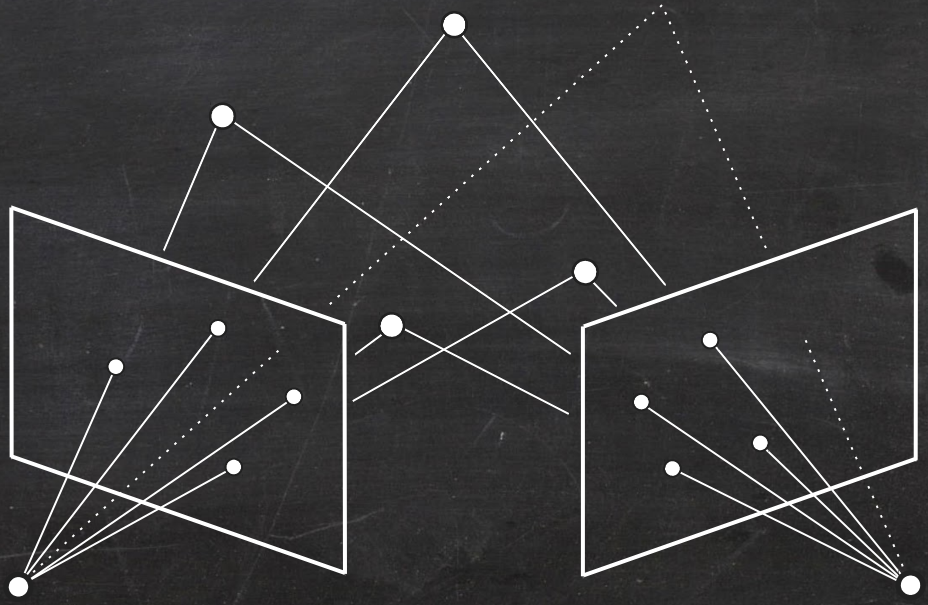
- Given: 2 images showing **5** points
- Goal: recover **5** points in 3D, and both (relative) camera poses



This problem has 20 solutions for generic input images
(counted over the complex numbers).

An Underconstrained Problem

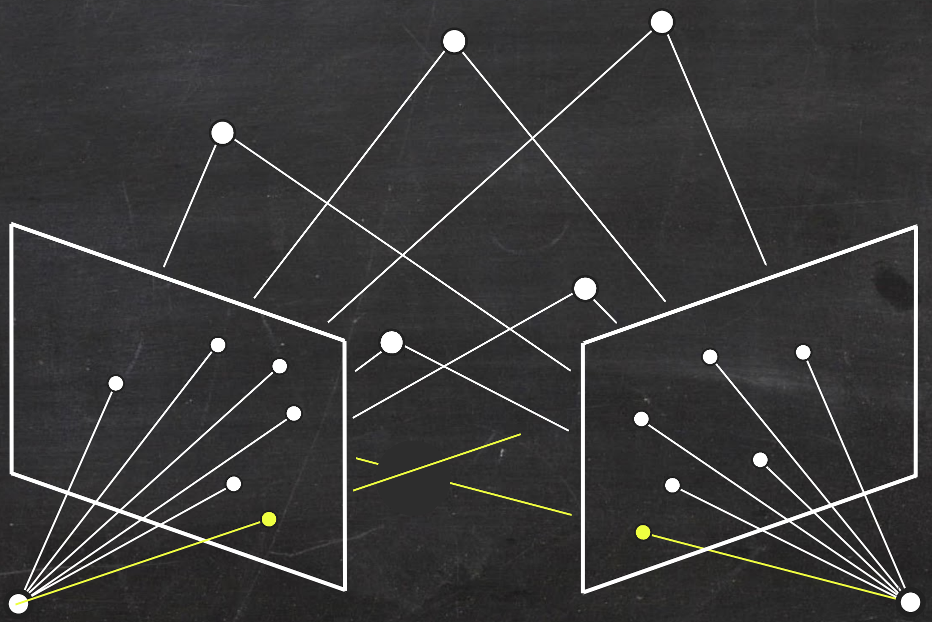
- Given: 2 images showing **4** points
- Goal: recover **4** points in 3D, and both (relative) camera poses



This problem has infinitely many solutions for generic input images.

An Overconstrained Problem

- Given: 2 images showing **6** points
- Goal: recover **6** points in 3D, and both (relative) camera poses



This problem has 0 solutions for generic input images.

Some input images have solutions, but they are **not stable under noise** in the input images!

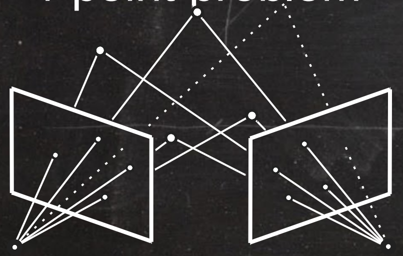
Minimal Problems

Definition: A 3D reconstruction problem is **minimal** if

$$0 < \# \text{ solutions} < \infty$$

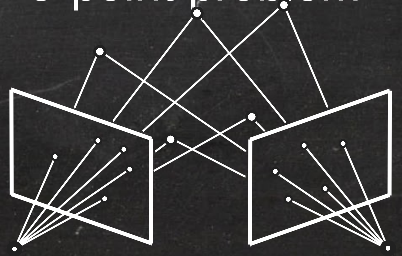
for generic (random) input images.

4-point problem



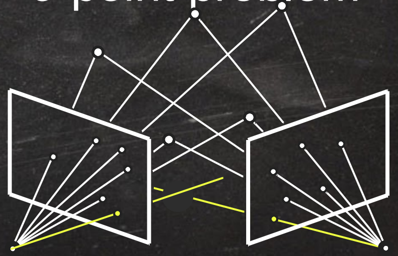
∞ solutions
not minimal

5-point problem



20 solutions
minimal

6-point problem

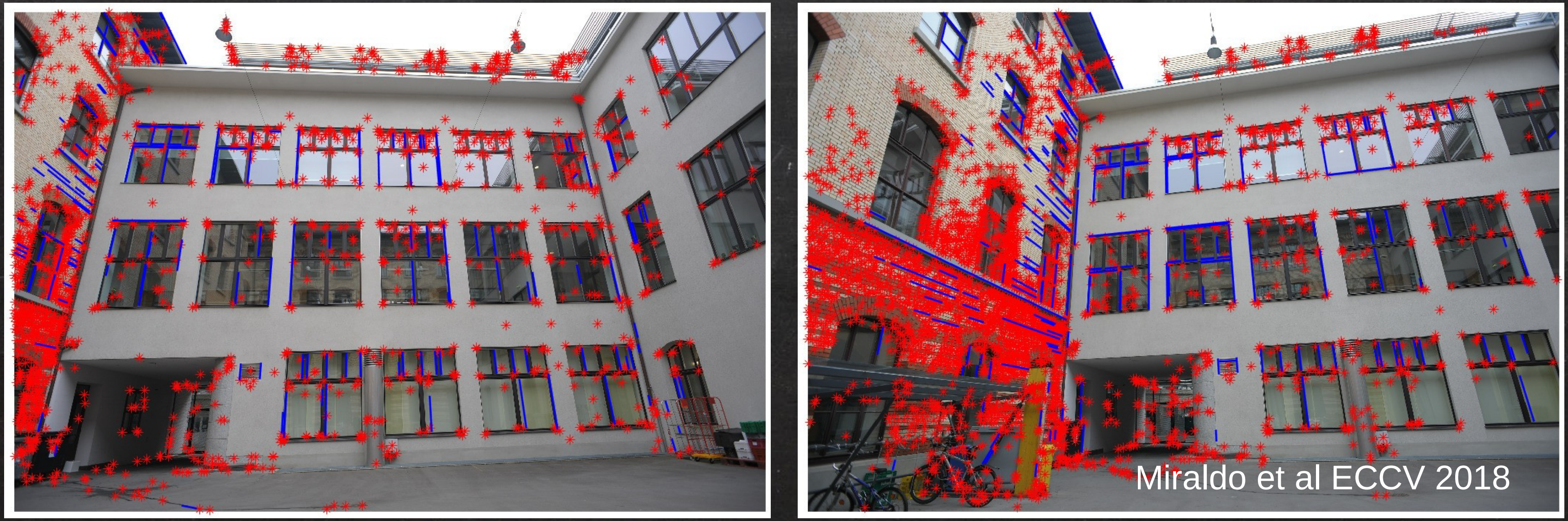


0 solutions
not minimal

Fundamental Research Questions

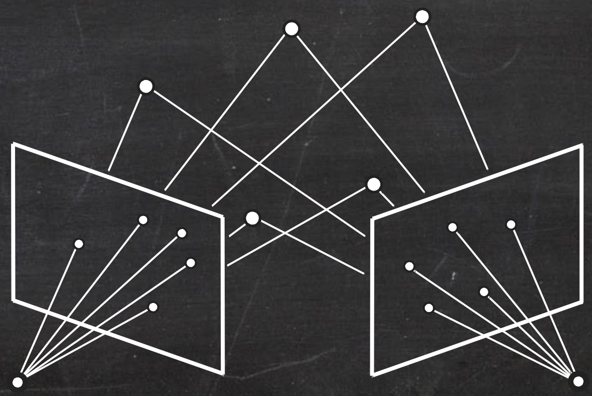
1. Can we list **all** minimal problems?
2. How many solutions do they have?

We do not only want to work with points,
but also with lines and their incidences!



Our Result

We provide the **first complete classification of all minimal problems** when all points and lines are visible in each given image.



Our Result

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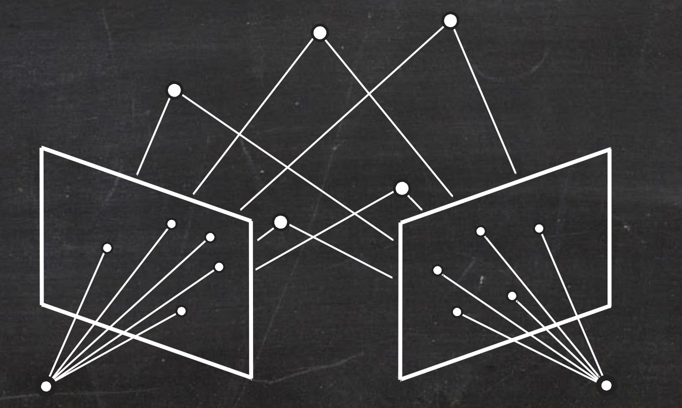
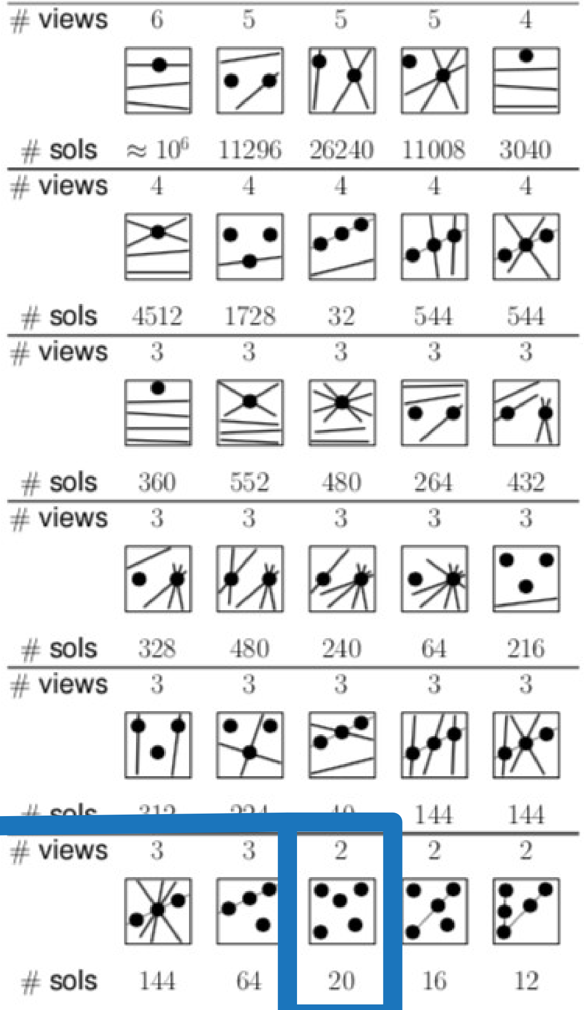


The 5-point problem has 20 solutions.

RESULT

There are **exactly 30 minimal problems** for *complete multi-view visibility* (modulo extra lines in 2 views).



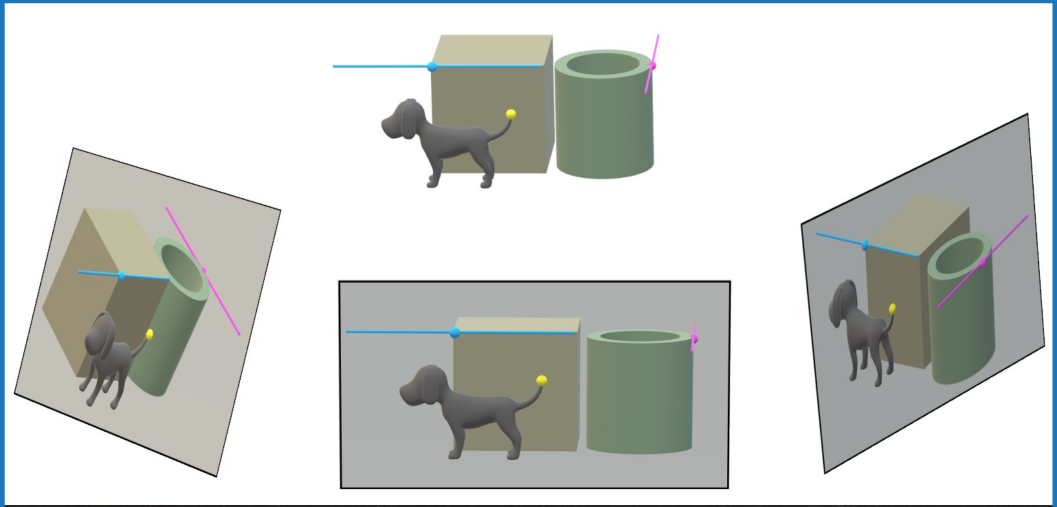
# views	6	5	5	5	4
# sols	≈ 10^6	11296	26240	11008	3040
# views	4	4	4	4	4
# sols	4512	1728	32	544	544
# views	3	3	3	3	3
# sols	360	552	480	264	432
# views	3	3	3	3	3
# sols	328	480	240	64	216
# views	3	3	3	3	3
# sols	312	224	40	144	144
# views	3	3	2	2	2
# sols	144	64	20	16	12

Our Result

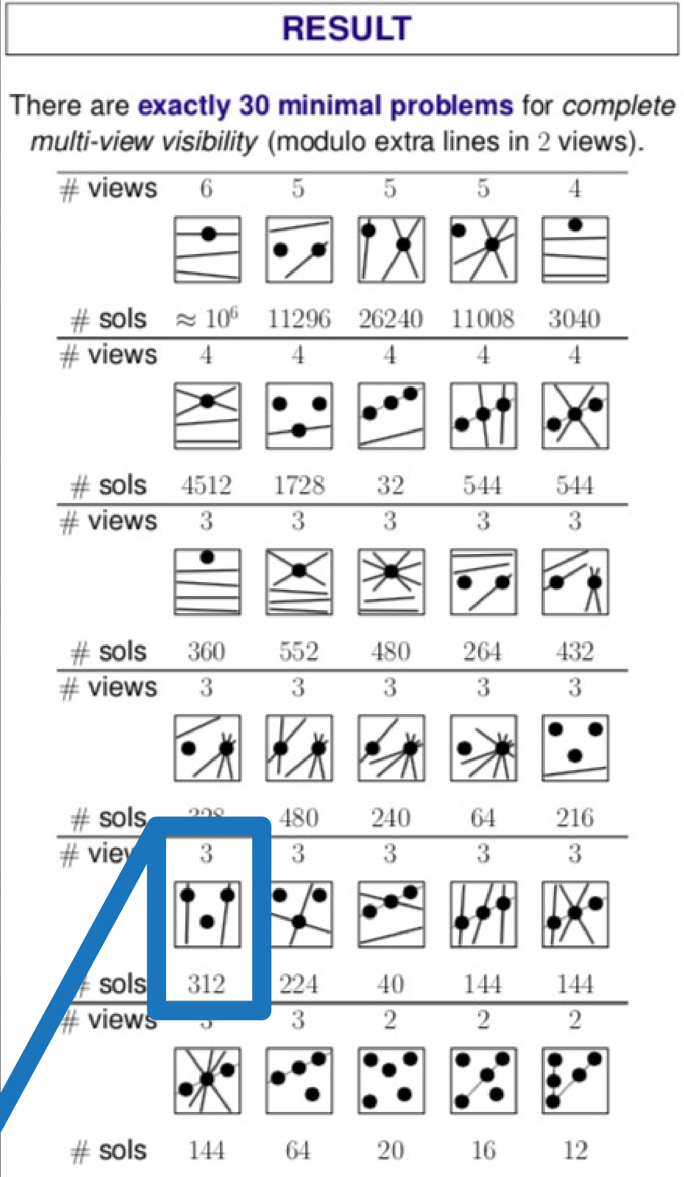
We provide the **first complete classification of all minimal problems** when all points and lines are visible in each given image.

First solver for such a high-degree problem based on state-of-the-art algorithms from **numerical algebraic geometry:**

TRPLP – Trifocal Relative Pose from Lines at Points, Fabbri et. al., CVPR 2020



This problem has 312 solutions
(counted over the complex numbers).

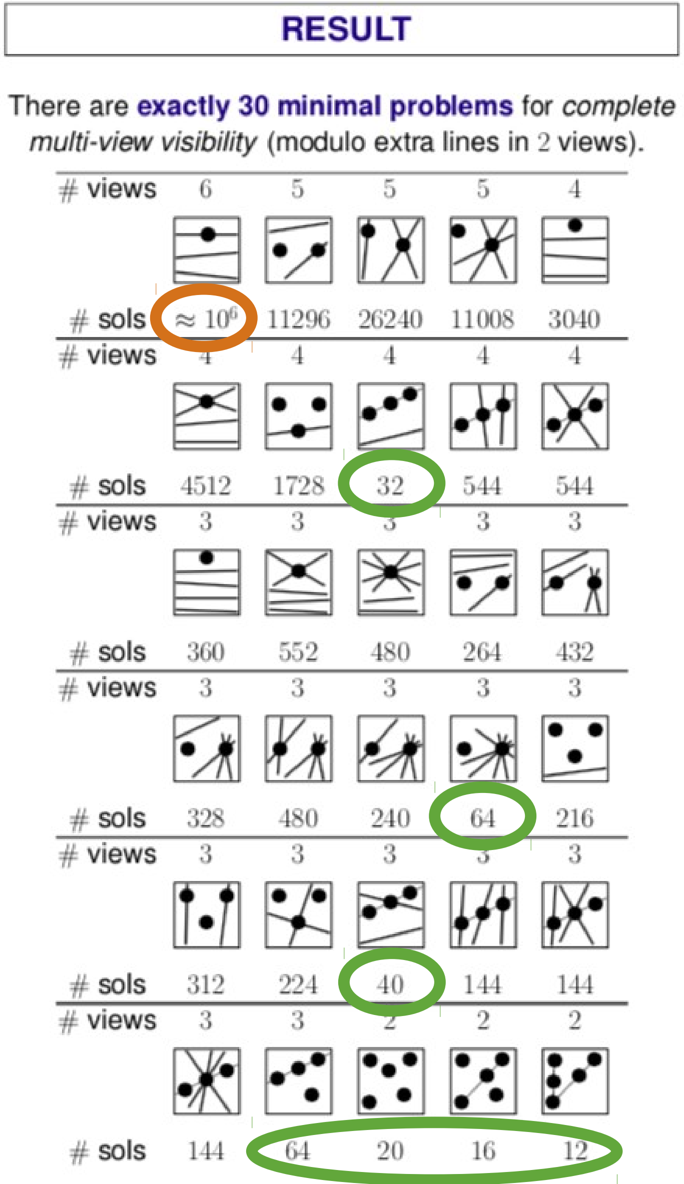


Our Result

We provide the **first complete classification of all minimal problems** when all points and lines are visible in each given image.

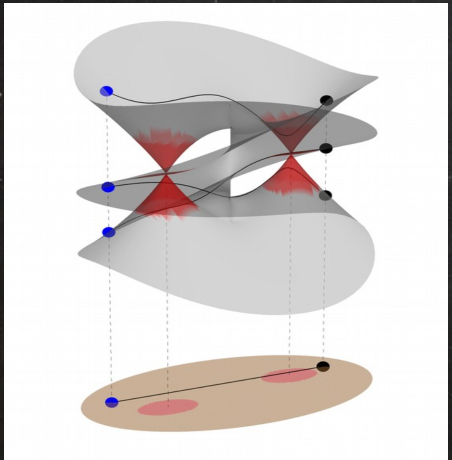
We **measure the complexity of each minimal problem** by computing its number of solutions

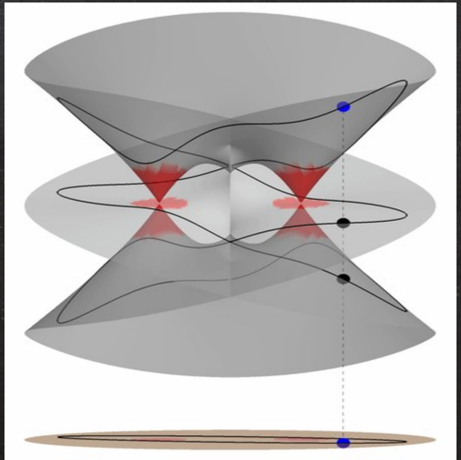
(counted over the complex numbers).

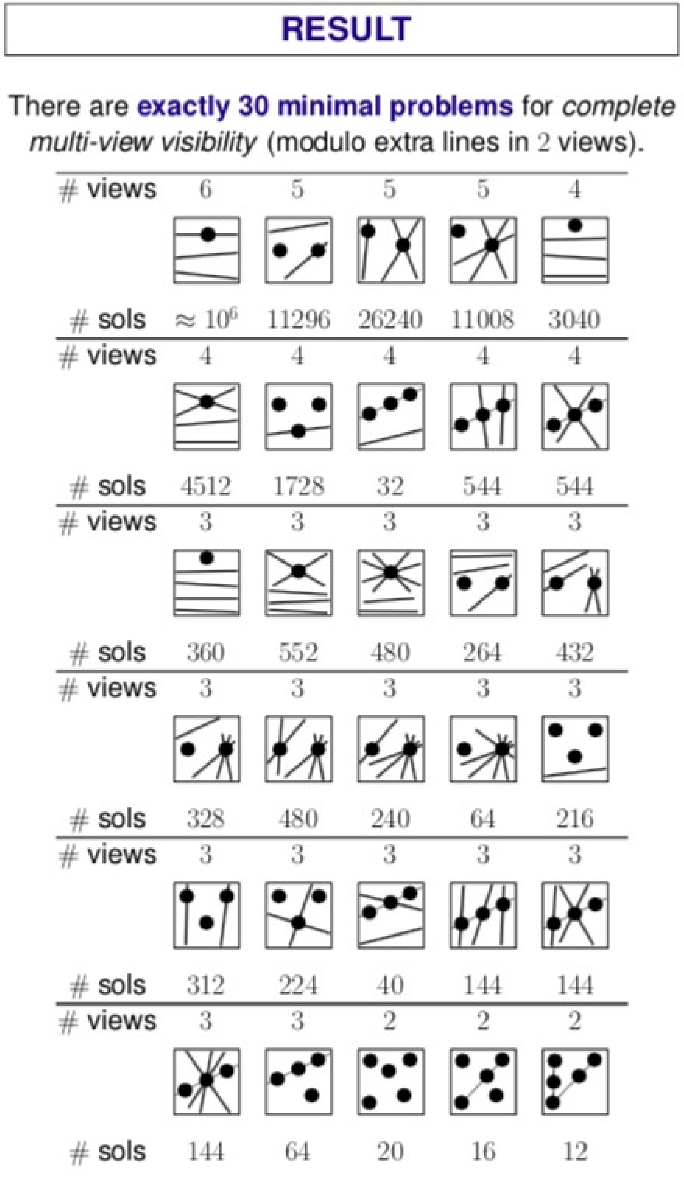


Our Tools: Nonlinear Algebra

- Algebraic geometry
  for proof of classification
- Gröbner bases
  symbolic computation of #sols
  for 2 & 3 views
- Homotopy continuation & monodromy
  numerical computation of #sols
  for 4, 5 & 6 views







RESULT

There are **exactly 30 minimal problems** for *complete multi-view visibility* (modulo extra lines in 2 views).

# views	6	5	5	5	4
# sols	$\approx 10^6$	11296	26240	11008	3040
# views	4	4	4	4	4
# sols	4512	1728	32	544	544
# views	3	3	3	3	3
# sols	360	552	480	264	432
# views	3	3	3	3	3
# sols	328	480	240	64	216
# views	3	3	3	3	3
# sols	312	224	40	144	144
# views	3	3	2	2	2
# sols	144	64	20	16	12

What about partial visibility?

There can be missing data / occlusions in the given images.

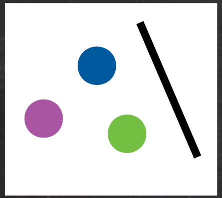


Image 1

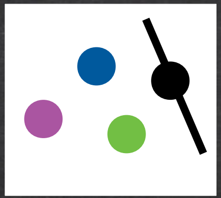


Image 2

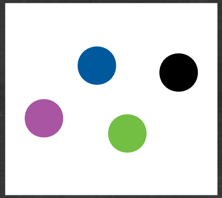


Image 3

- Minimal problems with complete visibility have at most 6 views.
  Minimal problems with partial visibility exists for arbitrarily many views!
  ⇒ Assume: **3 views**

- There are still ∞ minimal problems, and their classification is hard!
  ⇒ Assume: **each line is adjacent to at most 1 point**

⋯

There are still ∞ minimal problems!

Our Result

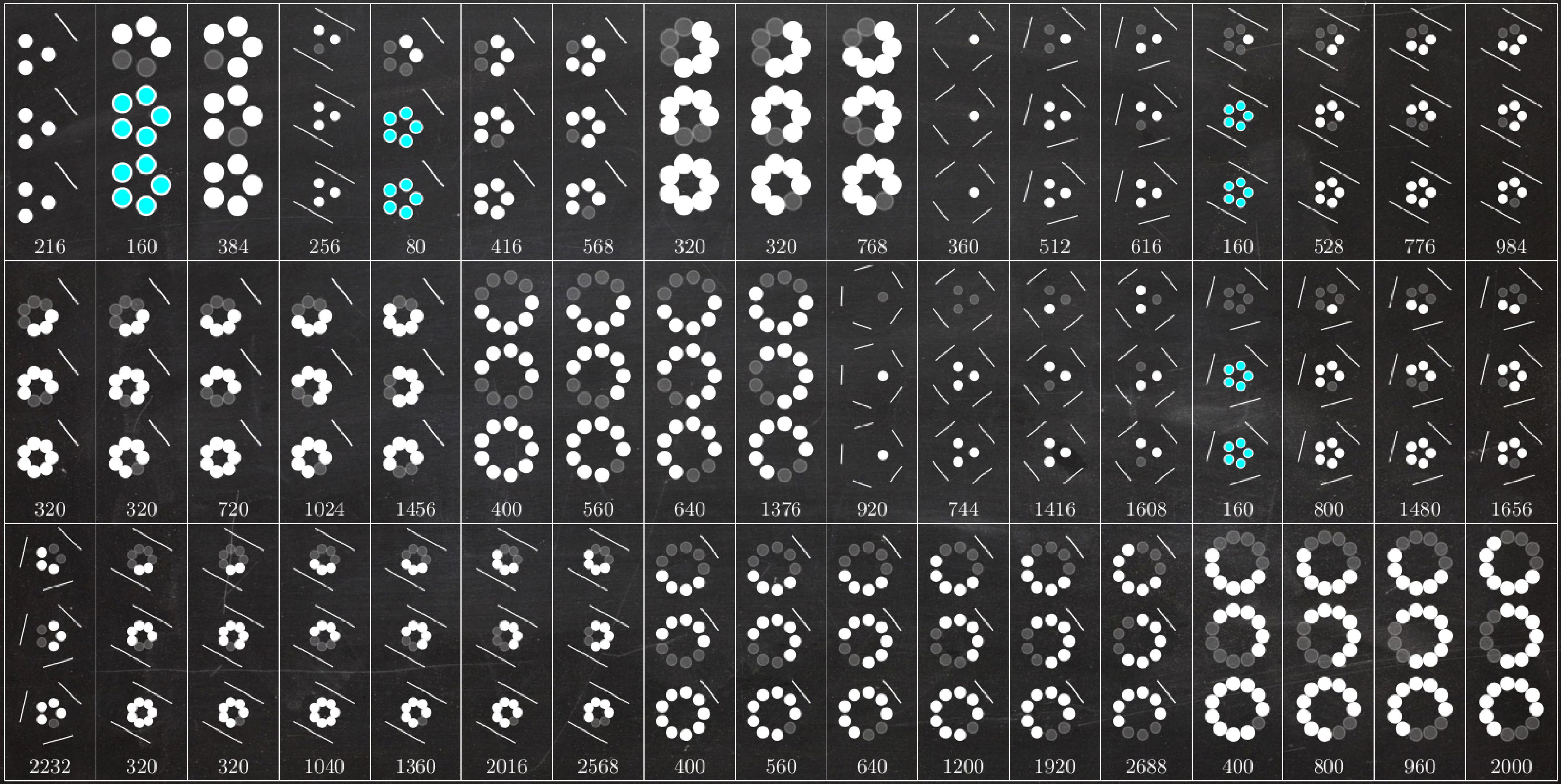
We **completely classify all minimal problems for 3 views** when each line is adjacent to at most 1 point:

There are **74575** equivalence classes of minimal problems.

Among them, **759** have **less than 300** solutions.

# solutions	64	80	144	160	216	224	240	256	264	272	288
# problems	13	9	3	547	7	2	159	2	2	11	4

There are **51** equivalence classes of minimal problems without incidences.



Final comment: Interaction between different sciences is key!

Thanks for your attention!