# Changing Views on Curves and Surfaces

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# Visual Event Surface

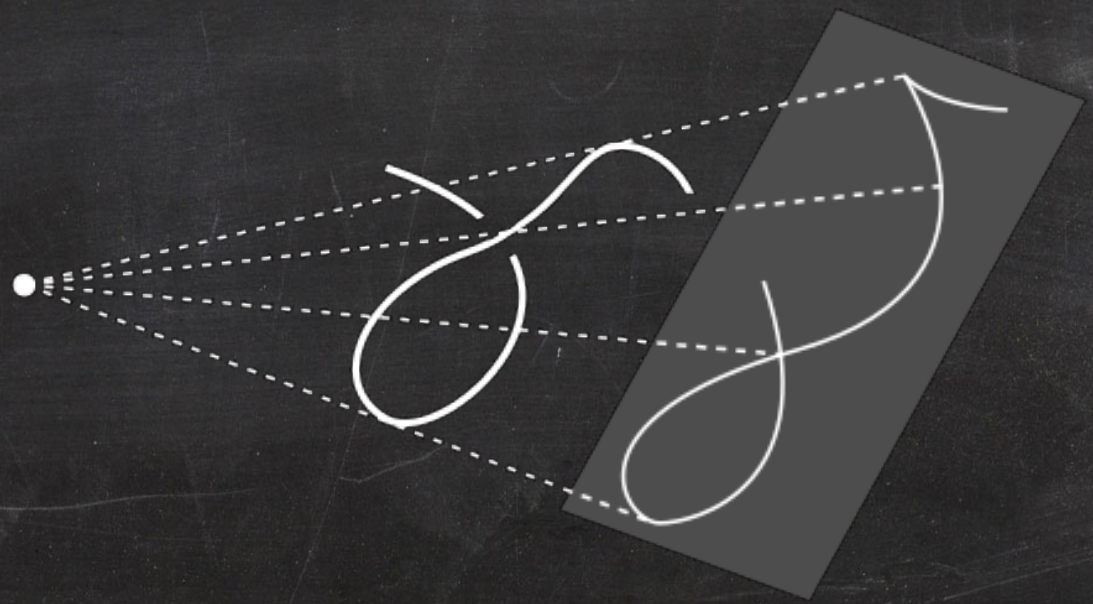
Consider a fixed curve or surface in 3-space.
Take pictures of that object with a moving camera.

At some camera points the image undergoes a qualitative change.
These points form the visual event surface.

# Section 1

## Curves

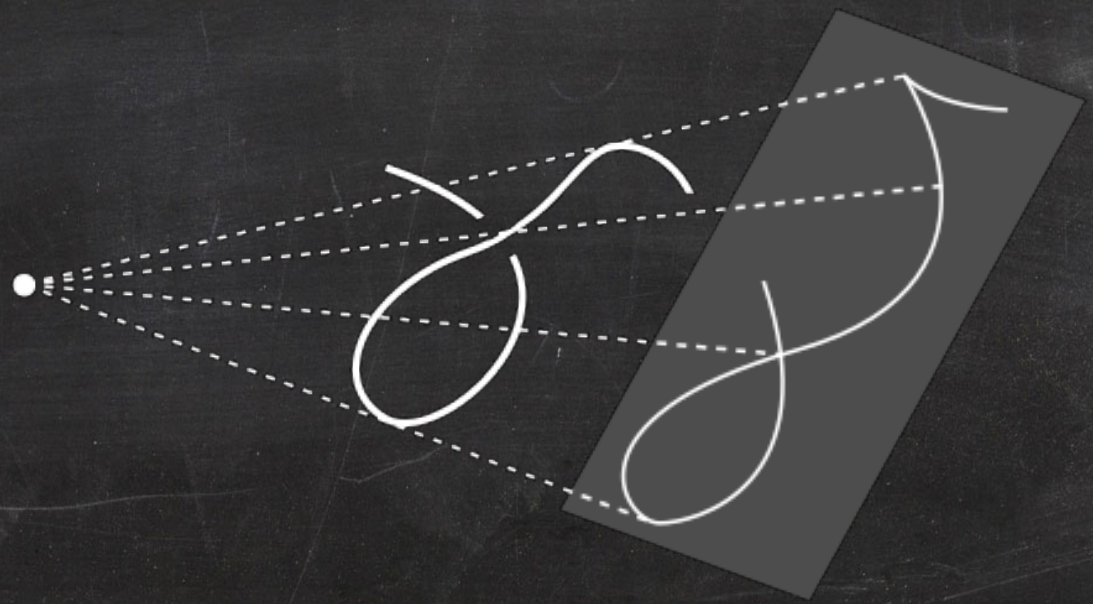
# Visual Event Surface



# Visual Event Surface

Consider a smooth curve in 3-space

- that is not contained in any plane, and
- has degree $d$ and genus $g$.



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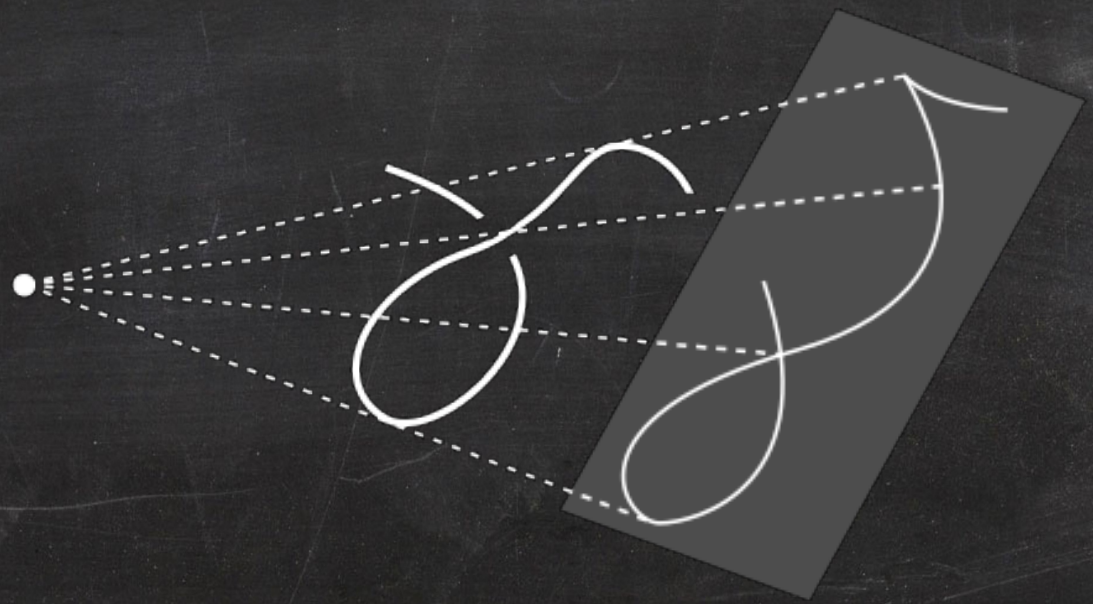
Projection from a general camera point yields a plane curve with $\frac{1}{2}(d-1)(d-2)-g$ nodes (over $\mathbb{C}$), and no other singularities.



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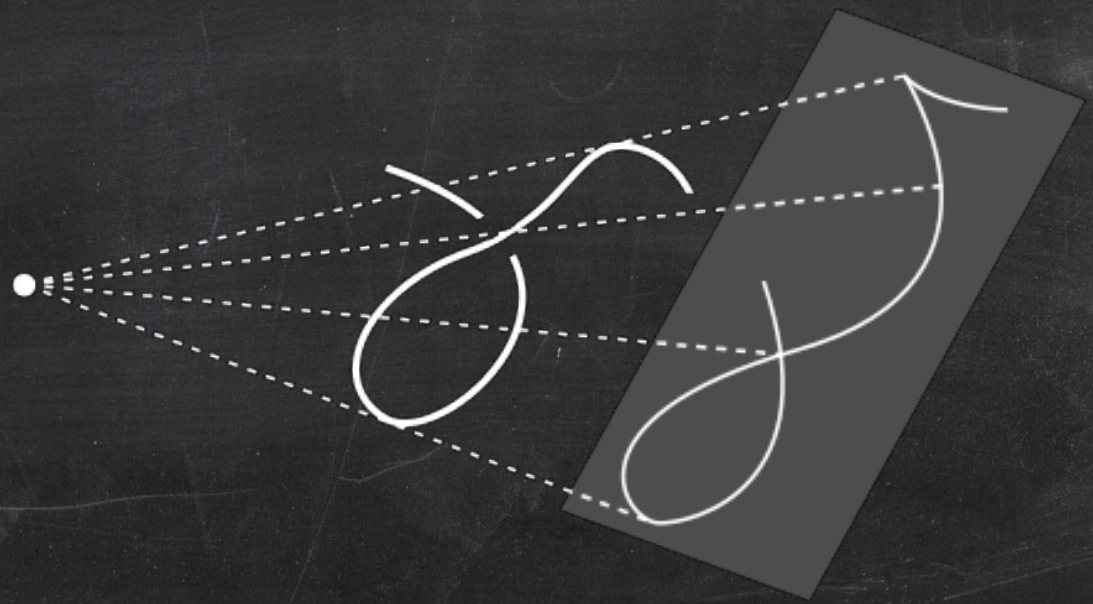
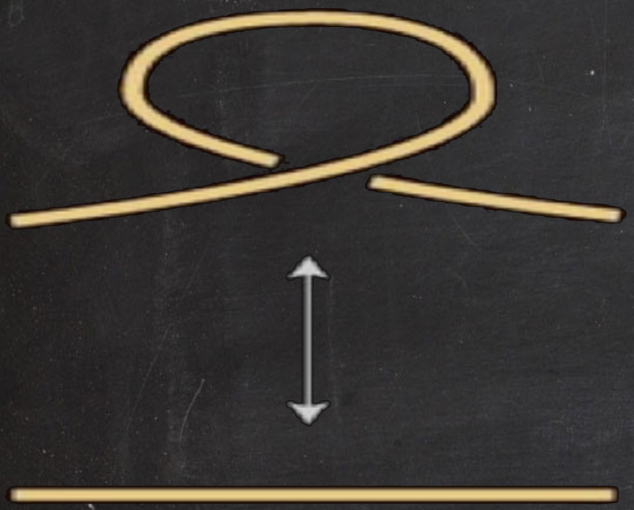


The visual event surface consists of those camera points where the plane curve has a different singularity structure.

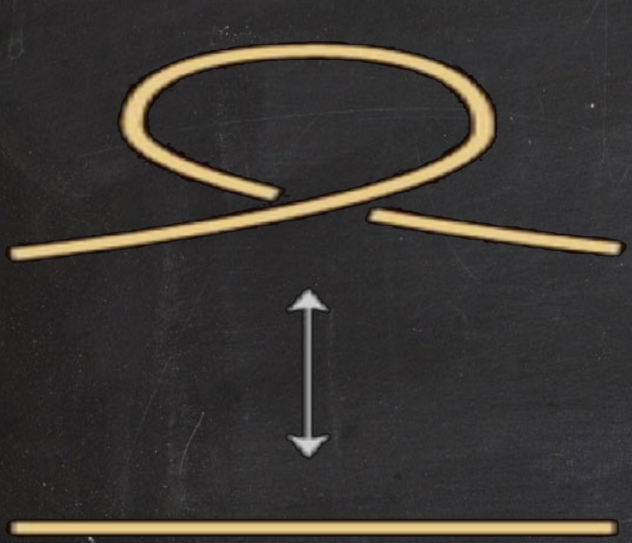
# Visual Event Surface: 3 Components



Tangential surface
union of all tangent lines
to the curve

$\rightsquigarrow$ cusp in image

# Visual Event Surface: 3 Components



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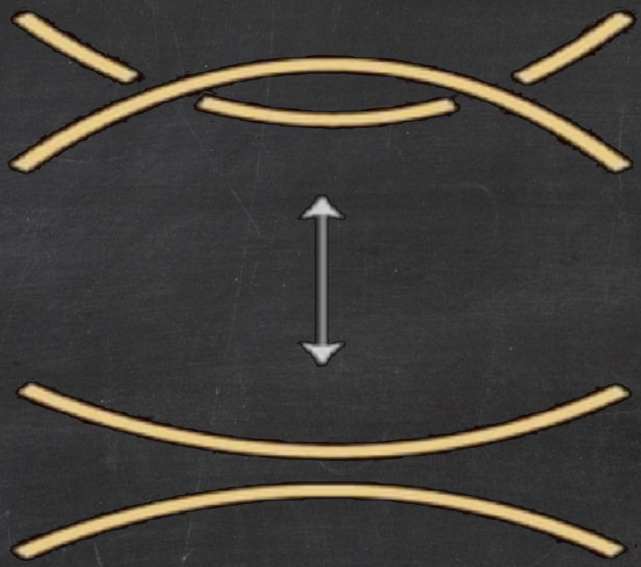
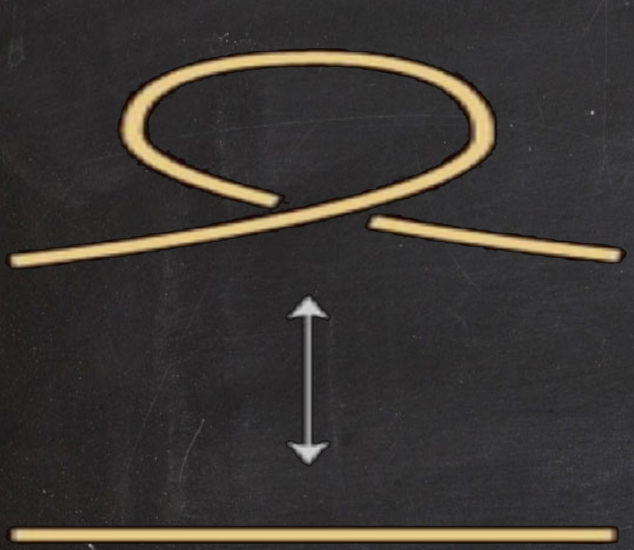


Edge surface
union of lines spanned by 2 points on curve whose tangent lines lie in a common plane

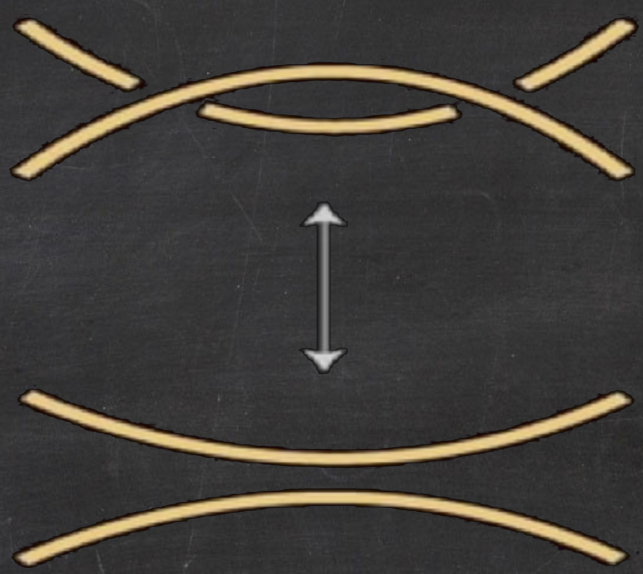
$\rightsquigarrow$ tacnode in image

# Visual Event Surface: 3 Components



Tangential surface
union of all tangent lines to the curve

$\rightsquigarrow$ cusp in image



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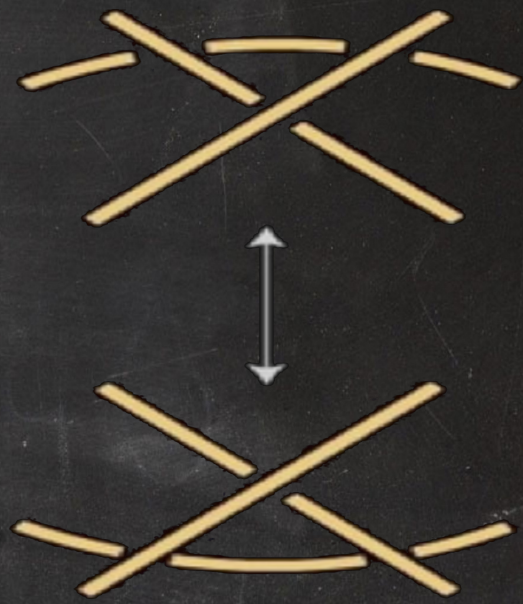


Trisecant surface
union of lines passing through 3 points on curve

$\rightsquigarrow$ triple point in image

# Degrees

For a general space curve $C$ of degree $d$ and genus $g$, the degrees of the compononents of its visual event surface are

$$\begin{aligned} \text{tangential surface} &: 2(d+g-1), \\ \text{edge surface} &: 2(d-3)(d+g-1), \\ \text{trisecant surface} &: \tfrac{(d-1)(d-2)(d-3)}{3} - (d-2)g. \end{aligned}$$

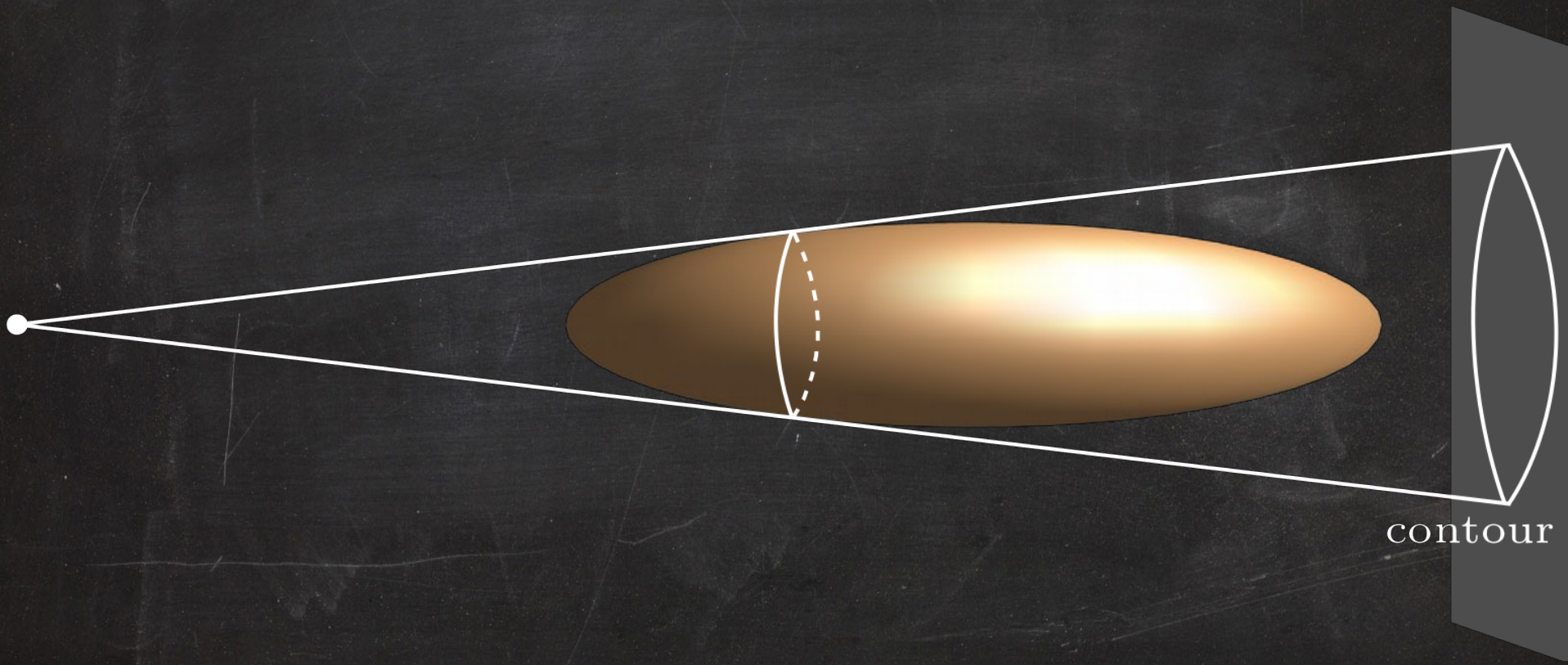
| $d$ | $g$ | tangential surface | edge surface | trisecant surface |
|---|---|---|---|---|
| 3 | 0 | 4 | 0 | 0 |
| 4 | 0 | 6 | 6 | 2 |
| 4 | 1 | 8 | 8 | 0 |
| 5 | 0 | 8 | 16 | 8 |
| 5 | 1 | 10 | 20 | 5 |
| 5 | 2 | 12 | 24 | 2 |
| 6 | 0 | 10 | 30 | 20 |
| 6 | 1 | 12 | 36 | 16 |
| 6 | 2 | 14 | 42 | 12 |
| 6 | 3 | 16 | 48 | 8 |
| 6 | 4 | 18 | 54 | 4 |

# Section 2

# Surfaces

# Visual Event Surface

Consider a general surface in 3-space of degree $d$.



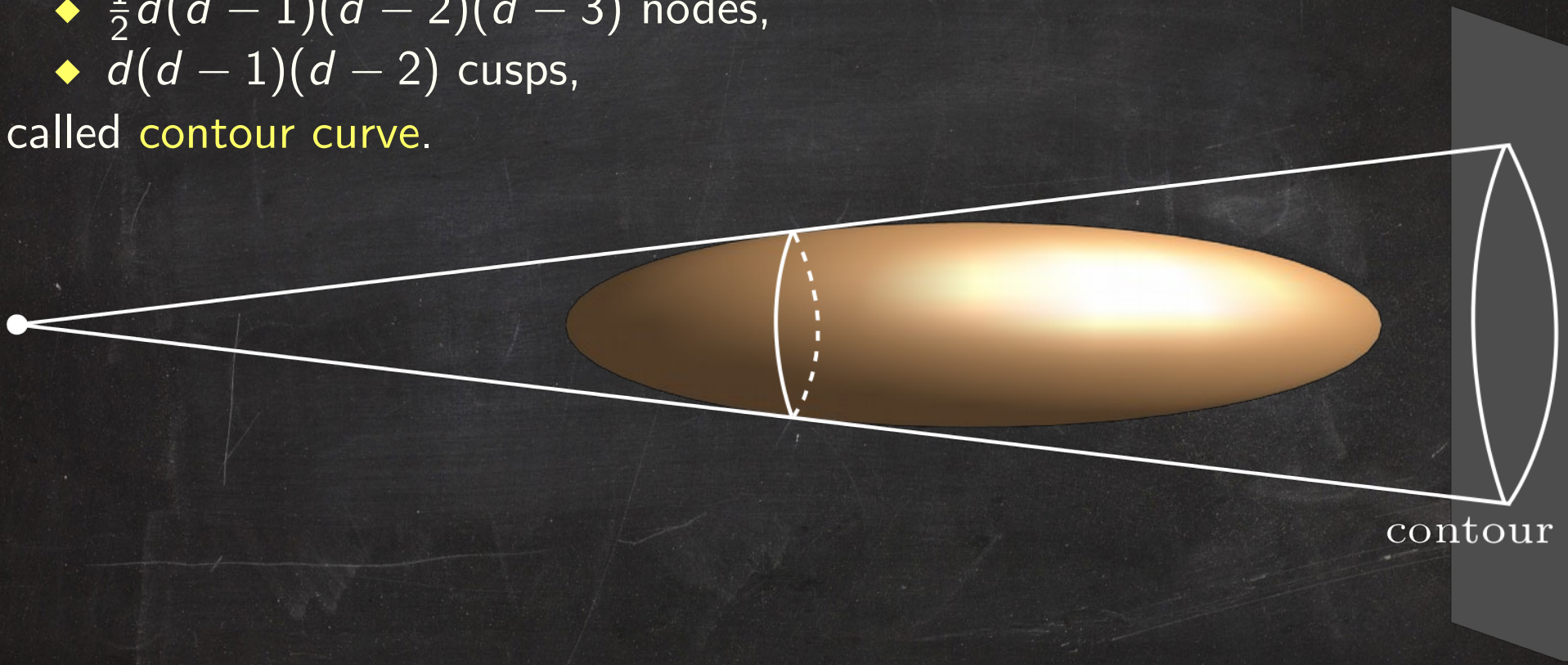
# Visual Event Surface

Consider a general surface in 3-space of degree $d$.

The branch locus of the projection from a general point is a plane curve with

- degree $d(d-1)$,
- $\frac{1}{2}d(d-1)(d-2)(d-3)$ nodes,
- $d(d-1)(d-2)$ cusps,

called contour curve.



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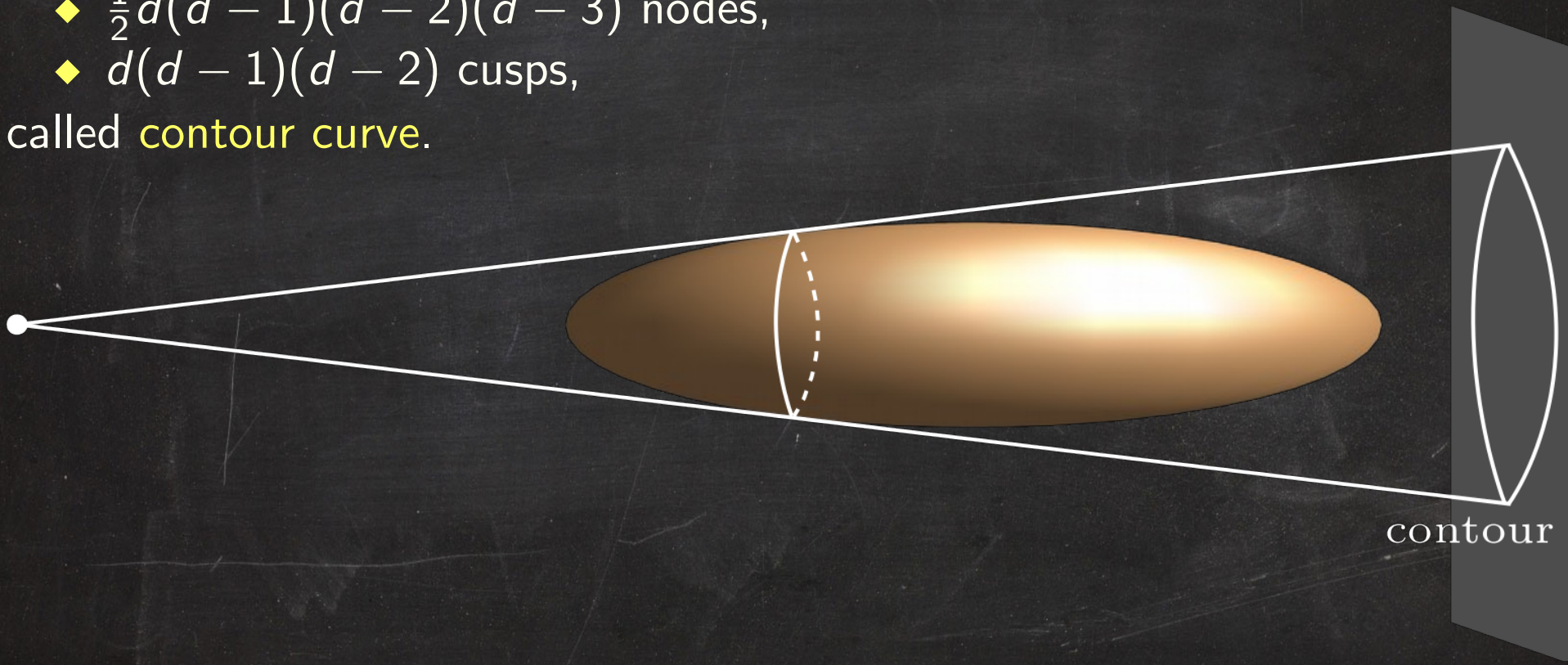
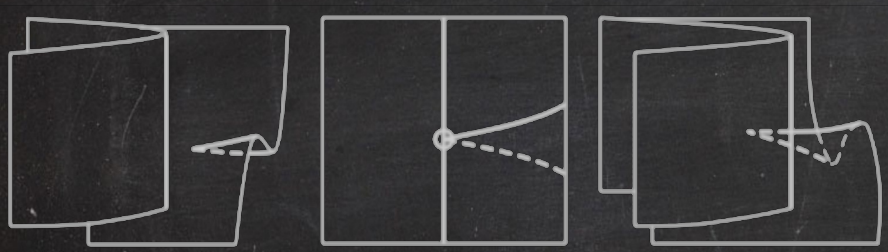


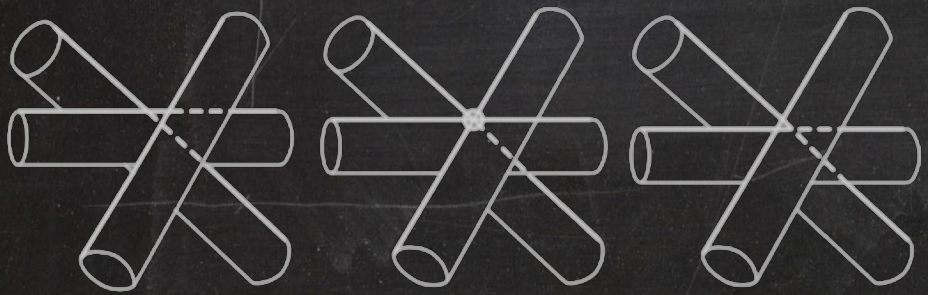

The visual event surface consists of those camera points where the contour curve has a different singularity structure.

# Visual Event Surface: 5 Components

Edge surface



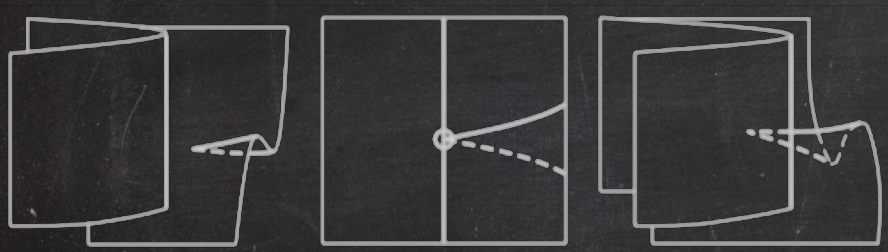
Cusp crossing surface



Tritangent surface

# Visual Event Surface: 5 Components

Edge surface



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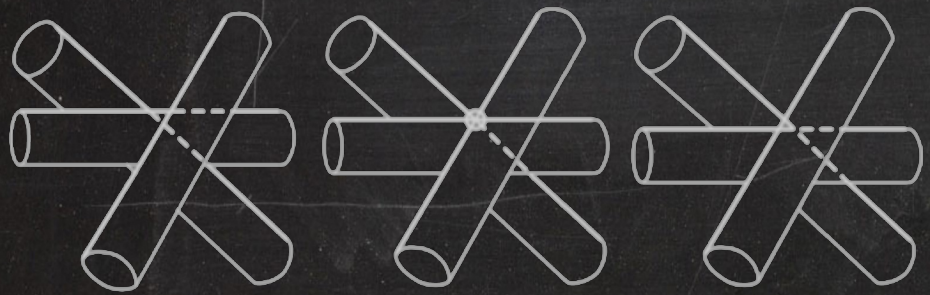
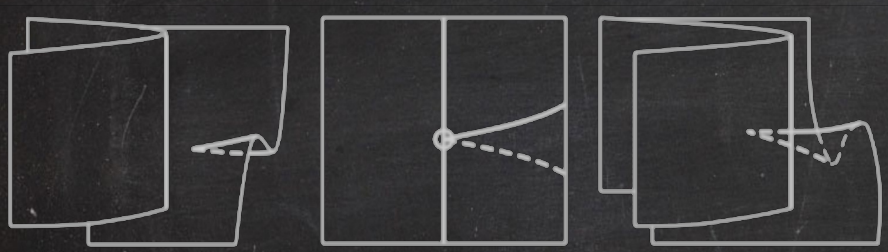


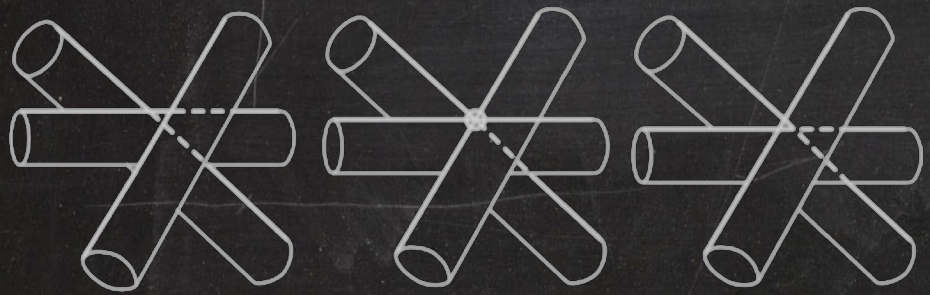
Tritangent surface
union of all tritangent lines to the surface

# Visual Event Surface: 5 Components

Edge surface



Cusp crossing surface
union of lines with contact of order $3+2$ at 2 points of the surface



Tritangent surface
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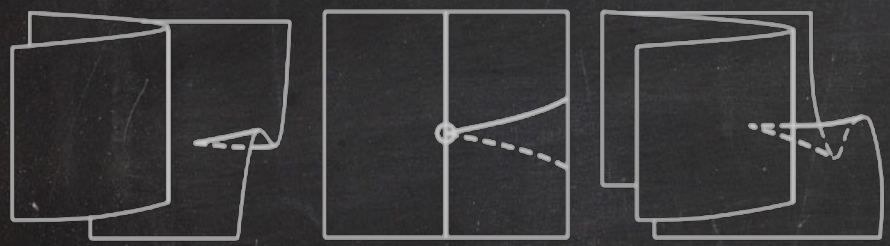
# Visual Event Surface: 5 Components

Edge surface
union of bitangent lines contained in bitangent planes



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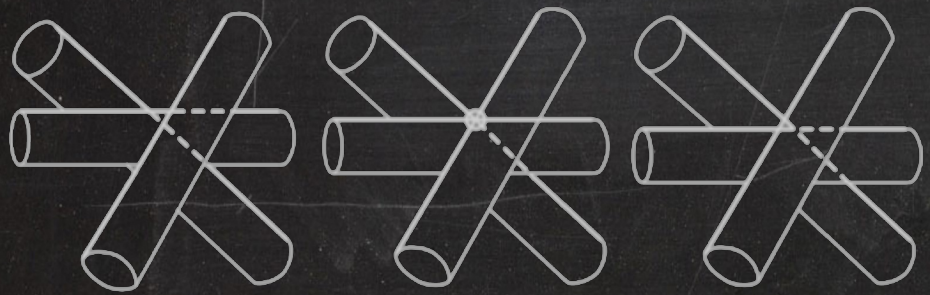

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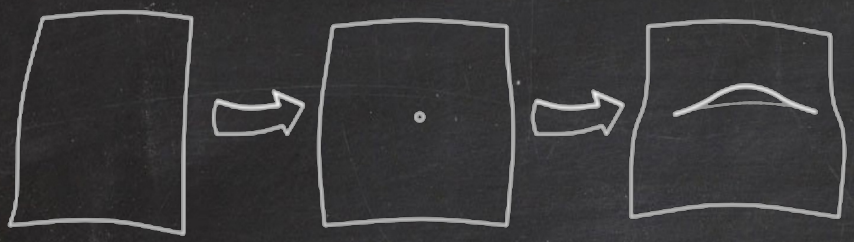
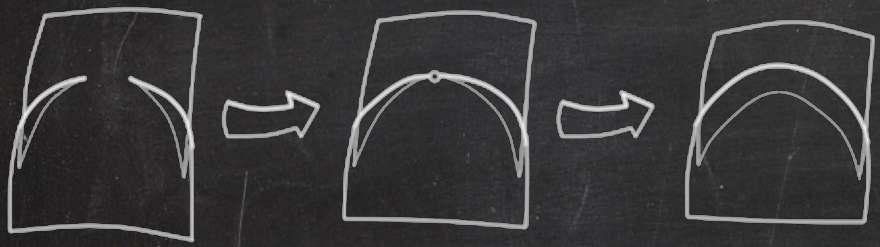


Parabolic surface



Over $\mathbb{R}$ there are 2 possible singularities in the contour curve.

Flecnodal surface

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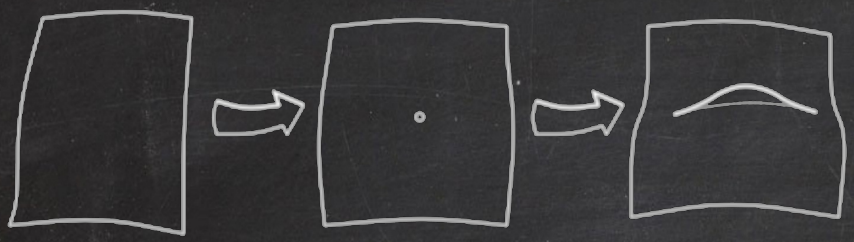

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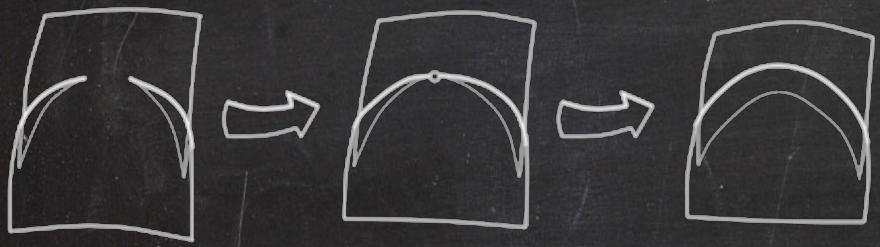

Over $\mathbb{R}$ there are 2 possible singularities in the contour curve.

Flecnodal surface
union of lines with contact of order 4 at a point of the surface

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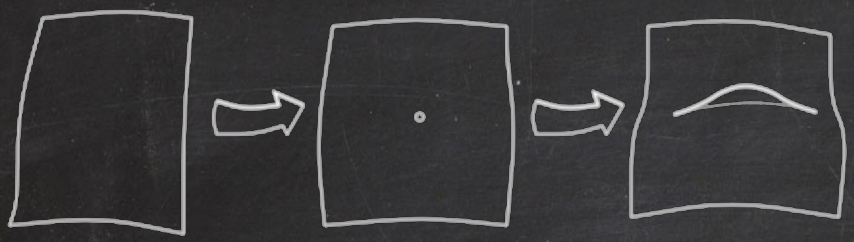
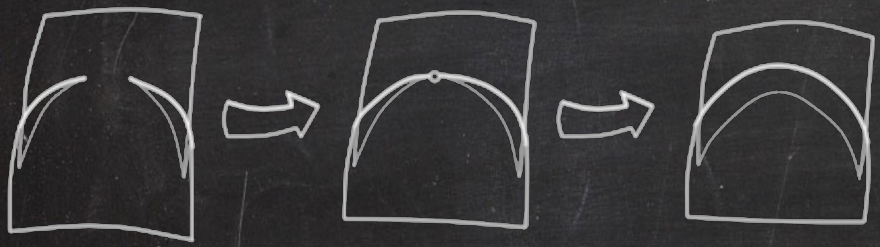


Parabolic surface

A general point on the surface has 2 lines with contact of order 3. A point is called parabolic if there is just 1 such line.



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Flecnodal surface

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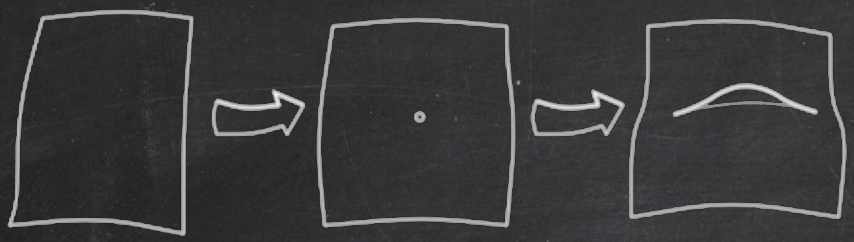

**Parabolic surface**
union of lines with contact of order 3 at a parabolic point of the surface

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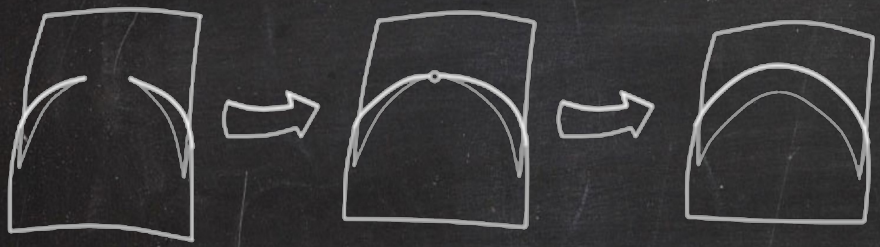

Over $\mathbb{R}$ there are 2 possible singularities in the contour curve.

**Flecnodal surface**
union of lines with contact of order 4 at a point of the surface

# Degrees

For a general surface $S$ in $\mathbb{P}^3$ of degree $d$, the degrees of the compononents of its visual event surface are

$$\begin{aligned}
\text{flecnodal surface} &: 2d(d-3)(3d-2),\\
\text{cusp crossing surface} &: d(d-3)(d-4)(d^2+6d-4),\\
\text{tritangent surface} &: \tfrac{1}{3}d(d-3)(d-4)(d-5)(d^2+3d-2),\\
\text{edge surface} &: d(d-2)(d-3)(d^2+2d-4),\\
\text{parabolic surface} &: 2d(d-2)(3d-4).
\end{aligned}$$

| $d$ | flecnodal | cusp crossing | tritangent | edge | parabolic |
|---|---|---|---|---|---|
| 3 | 0 | 0 | 0 | 0 | 30 |
| 4 | 80 | 0 | 0 | 160 | 128 |
| 5 | 260 | 510 | 0 | 930 | 330 |
| 6 | 576 | 2448 | 624 | 3168 | 672 |
| 7 | 1064 | 7308 | 3808 | 8260 | 1190 |


Thanks for your
attention