NCG Group New Results and Open Problems



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PoA for NSP

Simulator NP-completeness of SC Convergence speed of strong-PNSP Open Problems



Lemma

The PoA for the NSP is in $\Theta(\text{diam}(G))$, even when only reachable friendship situations are considered.

Average cost function

• Upper bound:

$$2|F| \le \sum_{v \in V} \sum_{u \in F(v)} d_G(u, v) = 2 \sum_{\{u, v\} \in F} d_G(u, v) \le 2|F| \operatorname{diam}(G)$$

 \Rightarrow worst-case PoA \leq diam(G)



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Lower bound:



Maximum cost function

• Upper bound:

$$|V_F| \leq \sum_{v \in V} \max_{u \in F(v)} d_G(u, v) \leq |V_F| \operatorname{diam}(G)$$

with
$$V_F := \{v \in V \mid F(v) \neq \emptyset\}$$

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• Lower bound:



non-reachable friendship example \Rightarrow PoA = $\Theta(\text{diam}(G))$

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PoA for NSP Simulator NP-completeness of SC ergence speed of strong-PNSP

Maximum cost function

Reachable friendship example?

Maximum cost function



Simulator NP-completeness of SC Convergence speed of strong-PNSP Open Problems

Maximum cost function

Case 2:



Simulator NP-completeness of SC Convergence speed of strong-PNSP Open Problems

Maximum cost function

Friendships:



Simulator NP-completeness of SC Convergence speed of strong-PNSP Open Problems

Maximum cost function



Simulator

NP-completeness of SC

Find best NE





Shortcut fixation:



Find social optimum

• 3-CNF-SAT reduction does not work

- Graph problems: Vertex Cover, Dominating Set, ... ~> dynamic edges?
- Knapsack Problem, Bin Packing, ...
 → code value v in a graph with size log(v)?
- Set Cover: same as 3-CNF-SAT (with few nodes)...
- Is it really NP-hard?
 - Seems related to Minimum Spanning Tree ~> solvable with greedy strategy?
 - No obvious formulation as matroid
 - Counterexample for greedy strategy!

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Convergence speed of strong-PNSP

Known bounds

Reminder: weak-PNSP with average cost function:

- At most $|F|(\operatorname{diam}(G) 1)$ improving moves until next NE
- Reachable friendship example for tightness in O-Notation

Strong-PNSP (with maximum cost function): Example with $\Theta(|V_F|^2 \operatorname{diam}(G))$

 \Rightarrow Convergence speed somewhere in between!

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- **Q** Reminder: sorted cost vector as potential function \rightsquigarrow At most $\begin{pmatrix} |V_F| + \operatorname{diam}(G) - 1 \\ |V_F| \end{pmatrix}$ improving moves until next NE
- \Rightarrow Convergence speed somewhere in between!



Start with easy graph (path) and easy friendships (path)

 \rightsquigarrow could give graph and friendship characterizations for convergence speed

Open Problems

Open Problems

- NP-completeness of SC_{OPT,dec}
- Convergence speed of strong-PNSP
- PoS
- Characterization of graphs/friendships for convergence/good NE
- Shortcut problem anyone?